MATHEMATICS METHODS

MAWA Semester 1 (Unit 3) Examination 2020
Calculator-free

Marking Key

Section One: Calculator-free (50 Marks)

Question 1(a) (2 marks)

Solution

$$f(x) = (3 + x^3)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(3 + x^{3)})^{\frac{-1}{2}}(3x^2)$$

$$= \frac{3x^2}{2\sqrt{3 + x^3}}$$

Mathematical behaviours	Mark
• applies chain rule	1
• obtains correct result	1

Question 1(b) (2 marks)

Solution

$$z = t^2 \cos(2t - 1)$$

$$\frac{dz}{dt} = \cos(2t-1) \times 2t + t^2 \times (-2)\sin(2t-1))$$

$$= 2t\cos(2t-1) - 2t^2 \sin(2t-1)$$

Mathematical behaviours	Marks
• differentiates cos term correctly	1
• applies product rule and states result	1

Question 1(c) (3 marks)

Solution

$$y = 5\sin(4x+3)$$

$$\frac{dy}{dx} = 5\cos(4x+3\ ^2 + 16 \times \left(5\sin(4x+3)\right)^2$$

$$= 400\cos^2(4x+3) + 400\sin^2(4x+3)$$

$$= 400(\cos^2(4x+3) + \sin^2(4x+3)) \qquad ...(*)$$

$$= 400 \qquad \because \cos^2(4x+3) + \sin^2(4x+3) = 1$$

Mathematical behaviours	Marks
• differentiates correctly	1
• substitutes and simplifies to (*)	1
• evaluates correctly, stating Pythagorean identity	1

Question 2(a) **(2 marks)**

Solution	
$f(x)=0 \Rightarrow x^3-12x=0$ $\Rightarrow x(x^2-1)=0$ $\Rightarrow x=0,\pm\sqrt{12}$	
Mathematical behaviours	**Marks**
• equates function to 0 and obtains $x=0$ • states $x=\pm\sqrt{12}$	1 1

Question 2(b) **(4 marks)**

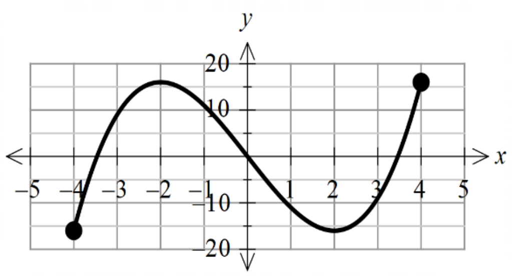
Solution	
$f(x)=x^3-12x$ $f'(x)=3x^2-12=0 \Rightarrow x=\pm 2$ $\quad f(2)=-16,\quad f''(2)=12>0 \Rightarrow$ min $f''(x)=6x$ $\quad f(-2)=16,\quad f''(-2)=-12 \Rightarrow$ max $f''(x)=0 \Rightarrow x=0,\ f(0)=0 \Rightarrow$ point of inflection	
Mathematical behaviours	**Marks**
• differentiates, equates to 0 and solves • obtains correct y values of the stationary points • uses second derivative test (or sign test) to determine nature of stationary points • locates point of inflection	1 1 1 1

Question 2(c) **(1 mark)**

Solution	
$f(x)=x^3-12x$ $f(-4)=-64+48=-16$ $f(4)=64-48=16$ $\therefore$ maximum is 16 since $f(-2)$ is also 16	
Mathematical behaviours	**Marks**
• determines $f(4)$ and concludes maximum	1

Question 2(d) **(3 marks)**

Solution



Mathematical behaviours	Marks
• plots zeros at 0 and such that $-4<x<-3$ and $3<x<4$ • plots stationary points and point of inflection accurately • obtains correct shape for the graph, scale and end points	1 1 1

Question 3(a) **(1 mark)**

Solution	
$x = 2$, $\frac{dc}{dx} = 2(8+1)^{\frac{1}{2}} = 6$	
Mathematical behaviours	Mark
• states correct answer	1

Question 3(b) **(4 marks)**

Solution	
$\int_0^2 x(2x^2+1)^{\frac{1}{2}}dx$ $= \frac{1}{4}\int_0^2 4x(2x^2+1)^{\frac{1}{2}}dx$ $= \frac{1}{4}\left[(2x^2+1)^{\frac{3}{2}}.\frac{2}{3}\right]_0^2$ $= \frac{1}{6}(27-1)$ $= \frac{13}{3}$	
Mathematical behaviours	Marks
• states the change as $\int_0^2 x(2x^2+1)^{\frac{1}{2}}dx$	1
• anti-differentiates correctly	1
• substitutes correct limits of integration	1
• determines correct answer	1

Question 4(a) **(2 marks)**

Solution	
$k + 3k + 5k + 4k = 1 \Rightarrow k = \frac{1}{13}$	
Mathematical behaviours	Marks
• states the sum of probabilities is 1	1
• deduces k value	1

Question 4(b) **(2 marks)**

Solution	
$P(X > 2) = 1 - \frac{1}{13} = \frac{12}{13}$	
Mathematical behaviours	Marks
• states an expression to calculate required probability	1
• determines probability	1

Question 4(c) **(2 marks)**

Solution
$P(X \leq 5 \mid X > 2) = \dfrac{\frac{8}{13}}{\frac{12}{13}} = \dfrac{8}{12} = \dfrac{2}{3}$

Mathematical behaviours	Marks
• writes fraction with the correct denominator	1
• obtains simplified result	1

Question 5(a) **(4 marks)**

Solution
(i)
$\int_0^{2\pi} 2\sin(4x)\,dx$
$= \left[\dfrac{-2\cos(4x)}{4}\right]_0^{2\pi}$
$= -\dfrac{1}{2}\left[\cos 8\pi - \cos 0\right]$
$= 0$
(ii)
$\int \dfrac{x+\sqrt{x}}{x}\,dx$
$= \int 1 + x^{\frac{-1}{2}}\,dx$
$= x + 2\sqrt{x} + c$

Mathematical behaviours	Marks
(i)	
• states anti-derivative	1
• evaluates result	1
(ii)	
• rewrites fraction as sum of two functions	1
• anit-differentiates including c	1

Question 5(b) **(3 marks)**

Solution
(i) $\int_1^6 f(x)\,dx = -\int_4^1 f(x)\,dx + \int_4^6 f(x)\,dx$ $= -1 + 4$ $= 3$ (ii) $\int_4^1 \left(2f(x)+1\right)\,dx = 2\int_4^1 f(x)\,dx + \int_4^1 1\,dx$ $= 2(1) + \left[x\right]_4^1$ $= 2 + (1-4)$ $= -1$

Mathematical behaviours	Marks
(i)	
• applies linearity of integrals, swaps bounds of integration and determines the correct result	1
(ii)	
• applies linearity of integrals correctly	1
• integrates correctly and calculates the result	1

Question 5(c) **(5 marks)**

Solution
$y = \frac{1}{e^{2x}+1} = \left(e^{2x}+1\right)^{-1}$ $\frac{dy}{dx} = \frac{-2e^{2x}}{\left(e^{2x}+1\right)^2} = -2\left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2$ $\int \frac{dy}{dx}\,dx = \int -2\left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2 dx$ $ie \quad y + C = -2\int\left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2 dx$ $ie \frac{1}{e^{2x}+1} + C = -2\int\left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2 dx$ $ie\left(\frac{-1}{2}\right)\frac{1}{e^{2x}+1} + C = \int\left(\frac{e^x}{\left(e^{2x}+1\right)}\right)^2 dx \Rightarrow A = \frac{-1}{2}$

Mathematical behaviours	Marks
• applies the chain rule to the derivative	1
• differentiates e^{2x} correctly	1
• recognises application of the Fundamental Theorem	1
• factors out -2 and re-writes fraction involving e^{2x} in numerator and denominator as one fraction squared	1
• multiplies both sides of expression by $-\frac{1}{2}$ to obtain desired result	1

Question 6(a) **(3 marks)**

Solution
$y = \frac{8x}{\left(x-1\right)^2} \Rightarrow \frac{dy}{dx} = \frac{\left(x-1\right)^2 8 - 8x \times (2)(x-1)}{\left(x-1\right)^4}$ $= \frac{8(x-1)-16x}{\left(x-1\right)^3} \Rightarrow c = 1,\ d = -1$ $= \frac{-8x-8}{\left(x-1\right)^3} \Rightarrow a = b = -8.$

Mathematical behaviours	Marks
• applies quotient rule	1
• differentiates both parts correctly and states the value of c and d	1
• simplifies result and states value of a and b	1

Question 6(b) **(2 marks)**

Solution	
Parallel to x axis $\Rightarrow \dfrac{dy}{dx}=0 \Rightarrow \dfrac{-8x-8}{(x-1)^3}=0 \Rightarrow x=-1 \Rightarrow y=\dfrac{-8}{4}=-2.$ So the coordinates of B are $(-1,-2)$	
Mathematical behaviours	**Marks**
• equates derivative to 0 and solves • states co-ordinates of B	1 1

Question 7(a) **(2 marks)**

Solution	
It is the area between the two curves from x = 0 to x = π.	
Mathematical behaviours	**Marks**
• states it is the area between the two given curves • states the area is from $x=0$ to $x=\pi$	1 1

Question 7(b) **(3 marks)**

Solution	
$\int \sin x - xe^{-x^2}\,dx = -\cos x - \left[-\dfrac{1}{2}\int -2xe^{-x^2}\,dx\right]$ $= -\cos x + \dfrac{1}{2}e^{-x^2} + c$	
Mathematical behaviours	**Marks**
• anti-differentiates $\sin x$ correctly • anti-differentiates xe^{-x^2} correctly • includes constant of integration	1 1 1